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A comparative study of queuing models in toll plaza

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Abstract

This paper analyze the collected data of a toll plaza before and after the implementation of FASTag using queuing theory by single server queuing model and multi-server queuing model and thus to find out if the implementation of FASTag has reduced the waiting time and increased the system efficiency.

Keywords: FASTag; Single-server model; Multi-server model

1. Introduction

Operational Research is the application of similar ideas to larger, more complex decisions that concern the operations of systems, such as businesses and networks of machines. Making these decisions using OR entails employing mathematical methods in order to solve a numerical version of the problem at hand.

Queuing theory is the mathematics of waiting lines.It is extremely useful in predicting and evaluating system. Queuing theory has been used for operations research, manufacturing and systems analysis. Traditional queuing theory problems refer to customers visiting a store, analogous to requests arriving at a device.

Toll plaza is a public or Private road for which people have to pay a fee to travel on that specific road. Tolls are often collected at toll plaza, toll gates, booth etc.,An observation was made in a particular Toll plaza with the Queuing models after and before the Implementation of FASTag in India. FASTag was Introduced in Oct 2017 by the Indian Government which is Implemented in around the year of 2020-2021 throughout the country, which reduce the waiting time of people in toll plaza.

This plan was Introduced under the scheme of Digital India.Under the system, a Radio Frequency Identification Device (RFID) Chip- Embedded sticker is put on the Vehicles allowing deduction of money automatically from the prepaid or savings account linked to it or directly toll owner.Which is now compulsory in India for vehicles passing Toll Plaza.

2. General concepts on queuing models

Queuing models are used to predict the performance of service systems when there is uncertainty in arrival and service times. However, since a wide variety of queuing systems can be encountered in practice, it is critical to understand the system under study in order to select the model that best describes the real situation. A great deal of thought is often required in this model selection procedure.

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2.1. M/M/1 Model

A single server M/M/1 queue where the inter-arrival times and service times are assumed to be exponentially distributed. The rate of arrivals & is fixed, regardless of the number in the system. The rate of the server u is fixed, regardless of the number in the system. Let denote the number of customers in the system.

2.2. M/D/1 Model

A single server M/D/1 queue where arrivals are Poisson with rate &, but the service rate is deterministic, ie., there is a constant service rate.

2.3. M/M/c Model

The multi-server M/M/c model where arrivals are Poisson with rate 2, there are c servers with c >= 1 and each server has an independently and identically distributed exponential service-time distribution with mean 1/μ.

Considering specific toll plaza working on NH-45 in Tamil Nadu. Which covers a stretch from Trichy to Dindigul with toll-able length of 87.273 Kms. Data collected from TD Toll Road Private Limited (TDTRPL). Data provided information of vehicles which crossed the lanes for a six hour time period from 6 am to 12 pm for two days (before implementation of FASTag and after implementation of FASTag). Using the data collected, let us now discuss the performing characteristics of the system.

2.3.1. Assumptions of the system

The system provide service to vehicles such as car/jeep, bus, multi axle vehicles, light commercial vehicles (LCV), truck, tractor, etc. The system has four lanes. Each server provides service independently. Service discipline followed by the system is First Come First Serve (FCFS). The system size and the population size are infinite. Hence the system can provide service to infinite number of vehicles. The arrival time of the vehicles in the four lanes from 6 am to 12 pm on a specified day when FASTag is not implemented was investigated similarly , after the implementation of FASTag also investigated .When the electronic toll collection is not practiced, the service rate is assumed to be exponentially distributed, where a server has to server on an average of four vehicles per minute. When FASTag is implemented, the service rate is deterministic with a constant service rate of 7.5 vehicles per minute.

3. Comparison of data using single server model

Let us now consider the toll booth as the queuing system with four lanes. Our aim is to study the following performance parameters of the system before and after implementation of FASTag. 1. Utilization factor or Busy period 2. Idle time of the system. Length of the queue 4. Length of the system (service + queue) 5. Waiting time in the queue 6. Waiting time in the system (service + queue) Consideration of M/M/L Queuing model The data collected on specific day (before implementation of FASTag) can be modelled as M/M/1 queuing model. By assuming that the arrival of vehicles follows Poisson distribution, since in non-overlapping interval of time, the arrival is random in each lane. The service provided is exponential since it is given that average of four vehicles to be served per minute.Consideration of M/D/1 Queuing model The data collected on specific date (after implementation of FASTag) can be modelled as M/D/1 queuing model. By assuming the arrival of vehicle follows Poisson distribution, since in non- overlapping interval of time, the arrival is random in each lane. Since FASTag is an electronic toll collection system, which is automatic service providing with a constant service rate of eight seconds per vehicle. Hence the service distribution is deterministic whereas in M/M/1 model it is probabilistic in nature. Let us consider the performing measures of both M/M/1 and M/D/1 Queuing model which is given by the following table:

Table 1 Performance measures of single server queuing model

Performing measures of the system	M/M/1 queuing model	M/D/1 queuing model
Arrival rate	Λ	λ
Service rate	μ	μ
Utilization factor ρ	$\frac{\lambda}{\mu}$	$\frac{\lambda}{\mu}$

Average idle time of the system $\boldsymbol{\rho_0}$	1-ρ	1-ρ
Expected number of customers in the system L	$\frac{\rho}{1-\rho}$	$\rho + \frac{\rho^2}{2(1-\rho)}$
Expected number of customers in the queue L_q	$\frac{\rho^2}{1-\rho}$	$\frac{\rho^2}{2(1-\rho)}$
Expected waiting time of customers in the system W	$\frac{L}{\lambda}$	$\frac{L}{\lambda}$
Expected waiting time of customers in the queue W_q	$\frac{L_q}{\lambda}$	$\frac{L_q}{\lambda}$

Using the above formulas, let us now compare the performance separately for each lane before implementation and after implementation of FASTag respectively.

3.1. Calculations of lane I

Table 2 Comparison in the time interval of one hour

Model	λ	μ	P	p_0	L	L_q	W in s	W_q in s
M/M/1	60	240	0.25	0.75	0.3334	0.0834	20	5
M/D/1	63	450	0.14	0.86	0.1514	0.0114	8.6512	0.6512

Hence, it is clearly shown from the above calculation M/D/1 model reduces the waiting time of the system from 20 seconds to 8.65 seconds. Also the busy period of the system is reduced from 25% to 14%.Similarly for various time interval upto 12 noon comparison for other lanes can be executed.

4. Comparison of data using multi server model

Let us now consider the toll booth with four lanes by the M/M/c model where c = 4 servers. Here the arrival of vehicles follow Poisson distribution and the service provided to the vehicles. are exponentially distributed. There are four lanes, where the four servers are providing service independently to each other. The system size and population size are infinite, thus the system can accommodate infinite number of vehicles. Hence the system is modeled as M/M/4 queuing model. Thus the performance measure of the system by M/M/c model with c = 4 is given by the following table:

Table 3 Performance measures of multi- server queuing model

Performing measures of the system	**M/M/c queuing model**
Arrival rate	λ
Service rate	μ
Number of servers	c
Utilization factor ρ	$\frac{\lambda}{c\mu} < 1$
Average idle time of the system $\boldsymbol{\rho_0}$	$\frac{1}{\sum_{n=0}^{c-1} \frac{(\frac{\lambda}{\mu})^n}{n!} + \frac{(\frac{\lambda}{\mu})^c}{c!} \frac{1}{1-\frac{\lambda}{c\mu}}}$
Expected number of customers in the system L	$\frac{(\frac{\lambda}{\mu})^c}{c!(1-\rho)^2} \rho \, p_0$
Expected number of customers in the queue L_q	$L_q + \frac{\lambda}{\mu}$

Expected waiting time of customers in the system W	$\frac{L}{\lambda}$
Expected waiting time of customers in the queue W_q	$\frac{L}{\lambda}$

Using the above formulas, let us now calculate the performance parameter of the system .

5. Observations

- Implementation of FASTag, it has been calculated that M/M/4 model has been utilised more by reducing the busy period of the system from 34.9% to 14.4%.
- Implementation of FASTag M/M/4 model has been utilised more by reducing the expected number of customers in queue from 0.0329 vehicles to 0.00034 vehicles(L_q).
- Implementation of FASTag M/M/4 model has been utilised more by reducing the expected number of customers in queue from 1.44 vehicles to 0.53 vehicles(L).
- Implementation of FASTag M/M/4 model has been utilised more by reducing the expected number of customers in queue from 0.352 seconds to 0.0052 seconds (W_q).
- Implementation of FASTag M/M/4 model has been utilised more by reducing the expected number of customers in queue from 154 seconds to 8 seconds (W).

5.1. Observation and result

- The busy Period is reduced by 16.07%.
- Expected length of the queue is reduced by 0.0154.
- Expected length of the system is reduced by 0.6581.
- Finally looking on the main factor, the waiting time of the queue reduced by 0.1811 sec the waiting time of the system reduced by 7.1811 sec.

6. Conclusion

It is now clear that implementation of FASTAG has increased the system efficiency by reducing the performance Parameter such as busy period of the system, length of the queue, length of the system, waiting time of the vehicles in the queue and waiting time of vehicle in the system.

Compliance with ethical standards

Disclosure of conflicts of interest

I have no conflicts of interest to declare.

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